RESEARCH BULLETIN 168

Cyclical Deficits and State General Fund Revenue Forecasts

A Case Study of Idaho, 1998-2010

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December 2007

University of Idaho
College of Agricultural and Life Sciences

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December 2007

Executive Summary This paper shows three things: 1) that the typical way of forecasting deficits can lead to policy choices that make them worse, 2) there is a better way to forecast revenue by focusing on the "long run" distribution, and 3) we can use the new approach to estimate how likely a deficit might be, and how much money would be needed if it occurred.

Published and distributed by the Idaho Agricultural Experiment Station, Gregory A. Bohach, Director. University of Idaho College of Agricultural and Life Sciences, Moscow, ID 83844-2337

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[1] The author would like to thank Mike Ferguson, Maureen Kilkenny and Chris Mcintosh for their insightful comments on earlier drafts of this manuscript. This research was funded by the Idaho Agricultural Experiment Station, project BDR253.

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Abstract

The State of Idaho has a General Fund revenue (GFR) cyclical deficit problem. Forecasting GFR with the *adding-up-of-revenue-streams* approach makes cyclical deficits worse by contributing to structural deficits. The *rule-of-thumb* and associated density forecasts can determine the likelihood of a point forecast occurring, clarify the existence and extent of the one-time surpluses or deficits, and help calculate the size of the budget stabilization fund needed (10 to 12% of GFR) to offset cyclical deficits. The *rule-of-thumb* forecasts provide a long-run trend in GFR that can be interpreted as the revealed preference for government services. The mean of a normally distributed *rule-of-thumb* forecast of GFR results in a stable forecast through business cycles with a smaller variance than the *adding-up* approach. The key parameter is the elasticity of real per capita growth in GFR to personal income. This elasticity equals 1.04 ($\pm$0.14) in Idaho from FY70 to FY06.

Key words

budget stabilization, business cycles, forecasting competitions, government forecasting, nonparametric methods, vector autoregression models

Cyclical Instability

State government officials in Idaho have a cyclical deficit problem. Yilin Hou defined three types of budget deficits: managerial, structural, and cyclical(Hou, 2006).[2] A cyclical deficit is defined as General Fund revenue (GFR) less than appropriated expenditures due to a downturn in the business cycle of the economy. The GFR deficits in Idaho between July 2001 and July 2003 were the result of three events. The first was an overly optimistic GFR forecast in December of 2000 for FY02, by both the Governor's and university economists. The *adding-up* approach of the Governor and university economists lead legislators to believe that revenue would be available to fund an unprecedented level of on-going expenditures. The second event was a national economic recession from March to November 2001 that created a cyclical deficit in Idaho by weakening the economy and reducing the flow of revenues. The recession was a result of a collapse in the capital gains bubble. The third was the personal and corporate income tax cuts in January of 2001 and 2002 that created a structural deficit by further reducing the state's ability to raise revenue sufficient to meet expenditures. The legislature compounded the problem of a cyclical deficit with a structural deficit.

The National Conference of State Legislatures lists revenue reliability, which includes stability, certainty, and sufficiency, as one of nine principles for a high-quality state revenue system. They define stability as low variability in tax revenues due to fluctuations in the business cycle(National Conference of State Legislatures, 1999). In 2001, Governing magazine analyzed all 50 states' tax structures and procedures and concluded that Idaho's revenue

[2] A managerial deficit is the result of cost overruns from unanticipated expenditures. A structural deficit takes place when either the statutorily defined tax rate or base is insufficient to meet the expenditure needs of the state.

process "is deficient when it comes to anticipating the future." The authors used as evidence the fact that the state's budget stabilization fund was too small at 2.2% of GFR, that some expenditure estimates were inaccurate, and that revenue estimates were predicted only one year in the future (Barrett, et al., 2001).

Hou argues that all states need at least three institutional structures to maintain continuity in public services through a cyclical deficit: 1) multiyear estimates of revenues, 2) multiyear estimate of expenditures, and 3) a budget stabilization fund sufficient to cover cyclical revenue shortfalls. Hou concludes that the development of multiyear revenue estimates depends, in part, on resolving the complexities associated with forecasting in an uncertain economic environment (Hou, 2006).[3] This paper addresses the revenue forecast portion of the cyclical deficit problem.

By way of background, each year Idaho legislators ask for nominal General Fund revenue forecasts 18 months prior to the end of the fiscal year from two groups of economists. One group is the Division of Financial Management economists in the Governor's office and the other is academic economists at three public universities in Idaho. Their specific charge is to provide their best judgment as to the actual amount of GFR that will be generated in the next fiscal year. The legislators use these two independent forecasts of revenue to set the overall level of expenditures for the next fiscal year. The legislature fills any subsequent gaps between

[3] In addition to concerns of the business cycle, state governments are also facing pressures from organizational and economic sources on revenues. Organizationally, Devolution and New Federalism place more responsibility for public services on the states, even as the costs of programs like Medicaid increase dramatically. Economically, consumer demand has shifted from goods to service sectors even as the competitive advantage of primary sectors has changed regionally and globally. These economic forces affect the size and stability of the states' sales and income tax bases

the actual revenues and expenditures with some combination of carryover, transfers, supplementals, and new taxes.

Both groups of economists use the same approach to forecast nominal GFR, which consists of *adding up* their forecasts of the individual revenue streams that flow into the General Fund. (For an example of this approach, see (State of Idaho Division of Financial Management, 2007b)). These revenue sources include personal and corporate income taxes, sales taxes, and fifteen other product and miscellaneous tax revenues that make up the General Fund in Idaho. These two groups of economists make their "*adding-up*" forecasts independent of the long-run trends in real GFR. They assume the long-run composite information is contained within the short-run individual trends of each tax revenue source. However, it is conceivable that legislators view the whole of GFR as the measure of citizens' willingness to fund public services and each source of tax revenue as simply one means to that end. By *adding up* each source of tax revenue, it is assumed that the sum of the parts will contain the properties of the whole. This assumption is subject to the fallacy of composition.

By using the short-run *adding-up* approach, the Governor's and university economists are largely silent on the topic of one-time budget surpluses or deficits.4 This is a conceptual rather than a technical problem because increasing the accuracy of the *adding-up* forecasts would not provide any more insight into the long-run trends in real GFR. In a short-run forecast, additional accuracy means better anticipating and incorporating the randomness due to the effects of past managerial, structural, and cyclical changes in the equations of their model or

[4] The adding-up approach does provide a measure of the growth in GFR, which when compared to previous growth rates can appear above or below average. However, the revenue growth due to inflation, population, and changes in the tax code complicate the interpretation of year-to-year growth in nominal GFR.

its results(Deschamps, 2004), (Rodgers and Joyce, 1996). “These sources of revenue are forecast using a combination of econometric methods and judgment” (State of Idaho Division of Financial Management, 2007b). In contrast, the short-term variability in GFR is the very effect that a long-run forecast tries to avoid. Mocan and Azad found that the GFR forecast error is reduced by using state-level economic trend data, and by not relying exclusively on judgment methods (Mocan and Azad, 1995). Bretschneider et al. found that simple regression models of GFR also increase forecast accuracy (Bretschneider, et al., 1989).

Ironically, even though legislators ask for a short-run forecast of nominal GFR, they immediately try to determine if a surge (or slowdown) of the growth in the GFR is permanent or temporary. If a significant portion of the growth in GFR is temporary, then limiting program expansion or increasing budget reserves may be prudent. If permanent, then tax cuts or program expansions may be appropriate. These radically different policy alternatives hinge fundamentally on knowing the difference between the short-term forecast and a long-term trend in General Fund revenues (Alt and Lowry, 2000). Absent this knowledge, it is possible for a legislature to make decisions that mistakenly result in changing a situation from bad to worse, as happened in Idaho in FY02. To prevent this situation from happening in the future, we are looking for a long-run trend in GFR.

In the forecasting process, the question arises for both analysts and legislators whether any particular estimate of GFR is “reasonable.” Determining reasonableness implies some standard of comparison, such as a long-run trend. At the margin, we believe that the growth in income drives the growth in General Fund revenue in Idaho. We hypothesize that an elasticity of the growth in real per capita GFR to personal income embodies both the long-run trend in GFR

and the revealed preference for funding public services in Idaho. As such, this elasticity can be used to develop two variants of a long-run forecast of GFR: the one a "*rule-of-thumb*" point forecast, and the other a density forecast. These long-run forecasts can serve as a standard against which to measure the magnitude of one-time surpluses or deficits implied by the *adding-up* forecast as well as the probability of an *adding-up* GFR forecast occurring. The goal is not to replace the *adding-up* forecasts but rather to provide a multiyear context for them. In the process, we can test the Bretschneider et al. hypothesis that GFR forecast accuracy increases with simpler regression models (Bretschneider, et al., 1989).

In the next section, we will develop a *rule-of-thumb* forecast of GFR as well as a transformation of this method into a density forecast of GFR. We will discuss the data needed to produce these forecasts. Next, we generate the results using these methods. Then we compare and contrast our *rule-of-thumb* and density forecast results with those from the *adding-up* approach. Finally, we summarize the findings and make some concluding comments.

Modeling GFR with Rule-of-thumb and Density Forecasts

An overview of the *rule-of-thumb* procedure to forecast GFR is as follows. 1) We estimate the elasticity of GFR to personal income. 2) with the help of the elasticity parameter and predictions about the growth in personal income, we forecast nominal GFR as the product of GFR in a reference year and its predicted rate of growth to a forecast year. 3) we duplicate step two repeatedly as a batch forecast of GFR in which we substitute an array of *i* reference years with their associated growth rates to the forecast year. The result is a normally distributed set of 36 forecasts for each of j forecast years. The mean of each distribution is a

point forecast we call the *rule-of-thumb* forecast. Finally, we can transform the probability distribution for each forecast year into a probability density function of GFR. By integrating across successive $50 million intervals of the function, we generate a density forecast of GFR. The density forecast reveals the likelihood of a given GFR point forecast actually occurring based on thirty-six years of experience.

There is a pattern in the personal income data that suggested a *rule of thumb* in estimating the growth in GFR. From FY1970 to FY2006, the pattern of the relationship between the real per capita growth in GFR and that of personal income appears to be 1:1 (see Figure 1). This pattern emerges even though the period includes six downturns in the U.S. economy(National Bureau of Economic Research, 2003).5 We can model the growth of an endogenous variable over time as the elasticity-weighted growth of its exogenous variable over time(Chiang and Wainwright, 2005) (see Appendix 1).

The *rule-of-thumb* fitted equation for real General Fund revenue is:

[5] Between FY70 and FY06, the six downturns in the U.S. business cycles were:
Dec. 1969 to Nov. 1970,
Nov. 1973 to March 1975,
Jan. 1980 to June 1980,
July 1981 to Nov. 1982,
July 1990 to March 1991, and
March 2001 to Nov. 2001
National Bureau of Economic Research. "Business cycle expansions and contractions." *National Bureau of Economic Research* (2003).

1) $$\frac{R_1}{R_i} = \left(\frac{Y_1}{Y_i}\right)^{e_{RY}} E$$

where R is sales tax rate adjusted, real per capita General Fund revenue,
Y is real per capita personal income,
e_{RY} is the elasticity of GFR to personal income,
E is the normally distributed error term,
subscript i is a reference year in the past, i = FY70 to FY06,
and 1 is the base period in the recent past, e.g., the average of values from FY02 throught FY06.

For a complete derivation of equation (1) see Appendices A1.1 and A1.2.

The *rule of thumb* forecast equation for real General Fund revenue is:

2) $$R_j = R_i\left(\frac{Y_j}{Y_i}\right)^{\hat{e}_{RY}}$$

where j is the forecast year, j = FY06 to FY10

The essence of a GFR forecast is the product of a base and a rate. In particular, a GFR forecast is the product of a given amount of GFR in the past (R_i) and the anticipated rate of GFR growth in the future $\left(Y_j/Y_i\right)^{\hat{e}_{RY}}$. A single *rule-of-thumb* forecast of real GFR is a product of three factors: a base and two complementary measures of the rate (see Appendix 1). The first factor is the base or the GFR amount in a reference year. A reference year is a past fiscal year between FY70 and FY06. The second factor is a rate, i.e., the growth in personal income between the reference year and the base period adjusted by the elasticity of growth in GFR to personal income.6 The third factor, which is also a rate, is the predicted growth in personal income as measured between the base period and the forecast year adjusted by elasticity. In the process of multiplying the two rates together, the base period term cancels out and the

[6] The term "base" is being used here in two different contexts. In the one, it is the amount of GFR in the past. In the other it is the average value of personal income over a time period.

result is a prediction of real GFR from the reference year to the forecast year (see equation 2). To transform the forecast of GFR from real to nominal, the product of the above three factors is multiplied by an index of the predicted growth in cpi, population, and sales tax rate. A complete derivation of the *rule-of-thumb* approach is available in Appendix 1.

The above procedure describes one forecast of GFR with a reference year base and a forecast year rate. By changing the reference year i from FY71 through FY06 and multiplying by its forecast year rate, we can repeat the above procedure in a batch estimation process for 36 forecasts of nominal GFR for each of j forecast years FY06 to FY10 (see Appendix 1 and Figure 3). The result is a forecast distribution for each forecast year. The mean of a distribution is the point forecast generated from a GFR base in the most representative reference year in the past 36 years. We call this point forecast the *rule-of-thumb* forecast. We can also use the forecast standard errors to calculate the confidence interval of the *rule-of-thumb* forecast (see equation 29).

To move from a point to a density forecast of GFR, we transform the forecast distribution into a probability density function (pdf) using the approach developed by (Silverman, 1986) (see Appendix 1). We create a density forecast of GFR by integrating across successive $50 million class intervals of the pdf and thereby generating a table of forecast probabilities by class interval, e.g., Table 3 (Tay and Wallis, 2000). The density forecast process converts a normal density function with an area under the bell-shaped curve approaching 100% (certainty) into a tabular histogram of probabilities that sums to approximately 100% across class intervals.

Data

The *rule-of-thumb* approach developed to forecast General Fund revenue in Idaho depends on six data sets (see Appendix 2). The data sets for Idaho include: 1) nominal GFR, 2) the portion of GFR funded by sales tax revenue, 3) personal income, 4) population, 5) the sales tax rate, and 6) the U.S. consumer price index(State of Idaho Division of Financial Management, 1970 - 2007a). There are 37 annual fiscal year observations in each series from FY70 through FY06. We deflated nominal GFR by the growth in CPI, population, and the sales tax rate to give real per capita GFR. We deflated GFR by the growth in the sales tax rate to account for an important source of structural change in Idaho's public finance system.7 We also deflated nominal personal income by CPI and population to derive real per capita personal income. Although forecasts of GFR are reported in nominal dollars, we deflate GFR and personal income to provide a common denominator for the econometric analysis and then translate our forecasts of real GFR back into nominal dollars (see Appendix 3 and Figure 1). The process of deflating and inflating GFR for population and the CPI carries with it the assumption that the Idaho legislature will increase revenues in a 1:1 ratio for changes in both inflation and population. Deflating for the consumer price index implies a desire by the legislature to maintain real levels of General Fund expenditures. Adjusting for population implies that the demand for public services such as schools, roads, prisons, health, and welfare services changes proportionately with population.

The *rule-of-thumb* forecasts of GFR depend on the forecast values of the independent variables. The forecast values of real personal income, inflation, and population were provided

[7] Michael Ferguson, chief economist in Idaho Division of Financial Management contributed this important insight.

by the State of Idaho and Global Insight(State of Idaho Division of Financial Management, 1970 - 2007a).

Results

A GFR forecast is the product of a base GFR amount in the past and the anticipated rate of GFR growth in the future. The *rule-of-thumb* forecast of the growth in real per capita GFR depends on knowing the elasticity of the GFR to personal income. We hypothesize that this elasticity parameter embodies the long-run revealed preference for funding public services in Idaho out of personal income. We test the relationship between GFR and personal income in several ways. A test of Granger causality with a four-year lag reveals that the growth in personal income causes the growth in GFR, but not the reverse. From this result, we can conclude that growth in real per capita GFR is an endogenous variable relative to the exogenous growth in real per capita personal income. Granger causality sustains a continued confidence in the elasticity-as-revealed-preference hypothesis.

We estimated the elasticity parameter by fitting the growth in GFR (in logs) to the growth in personal income (in logs) using the ordinary least squares (OLSQ) regression technique without an intercept term because the mathematics does not call for one (see equation (22)). Because this analysis uses time series data, we did an identification check of the OLSQ residuals for autocorrelation. We adjusted the residuals for autocorrelation by using both autoregressive (AR) and moving average (MA) terms lagged one period. The regression results indicate a stable parameter estimate of the elasticity coefficient (εgf) relative to the base period (see Table 1, and Figure 2). Over thirty-six years of observations, we can state with a 95% level of confidence that for every 1% growth in real personal income, we expect a 1.04 ± 0.14%

growth in real General Fund revenue.8 This 1:1 relationship in the growth in GFR to the growth in personal income again prevents us from rejecting our interpretation of elasticity as the long-run revealed preference for government services in Idaho.

The genesis of the name "*rule-of-thumb*" for this forecast approach comes from the observation that the growth in GFR in Idaho equals the sum of the growth in real per capita personal income, inflation, population, and sales tax rate. A simple *rule-of-thumb* forecast of GFR is the product of the most recent GFR and the growth in nominal total personal income to some point in the future. This simple forecast is dependent on the recent GFR (the base R_i) being representative. Since GFR fluctuates over the business cycle, the use of the most recent GFR as the reference year is problematic, especially at the turning points in the cycle. The *adding-up* approach has this representative-reference-year problem as well. The *rule-of-thumb* approach uses a batch forecasting procedure to overcome this problem.

A more complex but more accurate forecast of GFR is one that addresses the representative-reference-year problem. By applying the parameters of the fitted equation of GFR along with forecasts of real per capita personal income, inflation, and population growth in a batch estimation procedure, we can generate a probability distribution of GFR forecasts (see equation (27)). A *rule-of-thumb* forecast of GFR is defined as a point forecast at the mean of the probability distribution of GFR forecasts, e.g., see Figure 3. The mean of the forecast distribution solves the representative-reference-year problem by being associated with the most representative reference year statistically in the past 36 years (see bottom of Table 3). We

[8] The GFR to personal income elasticity for the State of Washington is 0.9 (Irv Lefberg, personal communication).

will show later that the batch estimation procedure is a key improvement of the *rule-of-thumb* over the *adding-up* approach.

A density forecast is based on a probability density function derived from the probability distribution of GFR forecasts. A density forecast is a tabular histogram that results from integrating across uniform intervals of a forecast probability density function (see top of Table 3). The advantage of a density forecast is the information-rich representation of the probabilistic nature of point forecasts by class interval (Tay and Wallis, 2000).

With the combination of the *rule-of-thumb* and density forecasts, we can analyze the point forecasts of GFR generated with the *adding-up* approach used by the Governor's and university economists. Recall that we could not falsify the hypothesis that the *rule-of-thumb* forecast is the long-run revealed preference for government services. This implies that the difference between the *rule-of-thumb* forecast and the actual GFR estimates the existence and extent of one-time surplus. With a density forecast we can make a subjective judgment about the likelihood of a point forecast occurring in the future given the observed random variability of GFR in the past. We use the density forecast in the form of a tabular histogram in $50 million increments to gauge the odds of the 18-month *adding-up* forecasts made by the Governor's and university economists.[9]

In FY06 and FY07, both the Governor's and university economists' 18-month forecasts of GFR had almost no chance of occurring from a density forecast perspective and underestimated the actual GFR by over $300 million in both years (see Table 3).

[9] The Governor's and university economists make two forecasts each January. The one is the initial prediction made 18 months before the end of the fiscal year (A). The other forecast is an updated prediction made half way through the fiscal year that ends June 31 (B).

Alternatively, if any of these GFR forecasts had occurred, they would have been extraordinary outcomes relative to those of the past 36 years. The Governor's 18-month GFR forecast for FY08 was about $20 million less than the *rule-of-thumb* forecast. It had about a 25% chance of occurring. By contrast, the university economist's forecast for FY08 is about $200 million above the *rule-of-thumb* forecast and has about a 2% chance of occurring. These results show that the combined *rule-of-thumb* and density forecasts offers a way to improve our understanding of *adding-up* point forecasts of General Fund revenue.

Because the *rule-of-thumb* forecasts are interpreted as long-run revealed preference amounts, the $110 and $75 million differences between the *rule-of-thumb* forecast and actual GFR in FY06 and FY07, respectively, are considered one-time surpluses.[10] The *rule-of-thumb* forecasts for FY09 and FY10 are $3,015 million and $3,255 million respectively. There are currently no *adding-up* forecasts available for comparison in FY09 and FY10.

The 95% confidence intervals for FY08 to FY10 are about ±10 to 12% of the *rule-of-thumb* forecasts of GFR. The confidence intervals around the forecasted level of GFR suggest a target range for the size of a budget stabilization fund between $290 and $350 million on about a $3 billion budget (see Tables 2 and 3). Currently, the State of Idaho has $291.1 million in three separate stabilization funds: Public Education Stabilization Fund ($110 million), a general purpose Stabilization Fund ($135.3 million) and an Economic Recovery Reserve Fund ($45.6 million)(State of Idaho Division of Financial Management, 2007a). If the $68 million currently in the Millennium Fund is included, then the total available for budget stabilization in Idaho

[10] A special session of the legislature in August 2006, put $100 million in carry over funds into a budget stabilization fund earmarked for school support.

is about $360 million. Idaho's stabilization fund is fully funded and adequate with only one chance in twenty there will not be enough money to cover a shortfall in revenue.

To judge the relative accuracy of the *rule-of-thumb* and the *adding-up* approaches, we compared hindcasts of GFR of these two approaches from (j) FY98 to FY06 (see Figure 4). Retrospectively, we can generate error vectors defined as the difference between these hindcasts and the actual GFR adjusted for subsequent tax law changes. Because GFR forecasts are made based on the tax structure as it exists at the time of the forecast, we altered the actual GFR by the amount subsequent tax code changes affected the revenue raised in that forecast year.11 We maintain the integrity of this experiment by re-estimating the elasticity εgf (0.98 ± 0.16) of the *rule-of-thumb* fitted equation and by using predicted personal income growth estimated 18 months before the end of the fiscal year. The *rule-of-thumb* hindcast used as reference years (i) FY71 to FY98 and a base period (1) equal to the average of GFR and personal income values in FY94 to FY98. We tested the mean and variance of the error vectors for significant differences.

Between FY98 and FY06, the mean of the *rule-of-thumb* error vector in forecasting GFR is $-39 million. The means of the university economists' and the Governor's *adding-up* error vectors are $-41 million and $-54 million respectively. The means as a percent of adjusted

[11] We adjusted the actual GFR for unanticipated tax structure changes that took place subsequent to the 18-month estimate. These changes include:
FY99 sales tax distribution for property tax relief ($54.7);
FY01 individual income tax temporary rate reduction (-$14), bracket indexing (-$2.4), estate tax to GFR ($7);
FY02 individual income tax temporary rate reduction (-$58.7), corporate income tax permanent rate reduction (-$6.8); FY03 sale tax temporary rate increase ($13);
FY04 sales tax temporary rate increase ($170.5), cigarette tax rate & distrib. change ($23.5), cash gift from Feds ($50);
FY05 agriculture personal property tax exemption (-$16.8). All figures in millions of dollars).
Amounts based on personal correspondence with Michael Ferguson, chief economist, Idaho Division of Financial Management.

actual GFR for these error vectors are 2.2%, 2.3%, and 3.0% respectively. There is no statistically significant difference in these three means over this period. Across the U.S. from FY96 to FY99, Hou reported average error rates in GFR forecasts ranged from 3.1% to 4.75%, with an average of 3.6%(Hou, 2006).12

Between FY98 and FY06, the standard deviation of the GFR *rule-of-thumb* error vector was about $140 million compared to $195 million for *adding-up* error vectors of both the university economists and the Governor.13 The GFR error vector standard deviation as a percent of adjusted actual GFR was 7.6% for the *rule of thumb* and 9.8% for the *adding up* approaches. The test of equality of the variance of the error vectors between the *rule-of-thumb* and either of the *adding-up* approaches is significant at the 67% confidence level (F test). From the variance results, we can conclude that in 2 of 3 forecast years the *rule-of-thumb* forecast will be closer to the adjusted actual GFR than the *adding-up* approach. Not only is the *rule-of-thumb* forecast a good predictor of the long-run GFR, but it is also a better predictor of GFR in the short-run than the *adding-up* approach is at least 67% of the time.

In FY00 and FY01, actual GFR was above the upper limit of 95% confidence intervals of the rule-of- thumb forecasts. This result, if it had been known, would have caused concerns that something extraordinary was happening in the Idaho economy. The 5% real per capita growth in GFR in those two years was being driven by something other than the 2% per year growth in personal income. Speculation at the time was that this surge in GFR was the result

[12] Division of Financial Management error rate between FY96 and FY99 was 3.06%, which drops to 1.44% when adjusted for unforeseen sales tax distribution changes (personal correspondence with Michael Ferguson, chief economist, Idaho Division of Financial Management).

[13] By adding another nine years of observation (FY89 to FY06), the average difference in the Governor's forecast of GFR is 3.8% below the actual forecast with a standard deviation of 8.6%.

of an increase in capital gains that appears in individual and corporate income tax returns but not in the data on personal income(State of Idaho. Division of Financial Management, 2001). 14

In 2000, Idaho's public finances were abundant. In January 2000, the legislature authorized $153 million in transfers to eleven different funds. They also approved a temporary income tax reduction of $16 million. In July, at the end of the state's fiscal year, there was a $185 million carryover into FY02 (State of Idaho Division of Financial Management, 2002). In December 2000, the Governor's and university economists forecast GFR of about $2,050 million for FY02, which would have been the third year in a row in which GFR was above the 95% confidence interval of the *rule of thumb* forecast distribution (see Figure 4).

In January 2001, after two years of surplus and an optimistic revenue forecast for FY02, an emboldened legislature increased expenditures and made the temporary cuts in personal and corporate income tax rates both larger and permanent. The $65 million tax reduction took effect during the 2001 recession that began in March. Because of the resulting $525 million swing in revenue between FY01 and FY02, some have argued that the FY02 General Fund revenue forecast was the most difficult to predict over this period. For FY02, the *rule-of-thumb* forecast was $1,780 million and the adjusted-actual GFR was $1,765 million. This $15 million error in the *rule of thumb* forecast compares favorably with the $300 million and $275 million errors by the university economists and the Governor respectively.

The years 2001 and 2002 were as bad as 2000 had been good. In January 2001, as mentioned above, the legislature provided $65 million in permanent personal and corporate income tax

[14] For example, capital gains can change income tax returns without changing personal income as currently measured.

reductions. The 2001 recession began in March and lasted until November. In July of 2001, the Governor began ordering a series of holdbacks totaling $65 million. Terrorists attacked New York and Washington in September. By the end of November, despite the holdbacks Idaho had spent all of the $184 million carryover as well. In January of 2002, the legislature authorized $96 million in transfers into the General Fund from various ad hoc stabilization funds. From January 2001 to January 2002, the GFR went from a $180 million surplus to a $345 million deficit all within the context of a $2 billion budget. A $200 million GFR deficit continued into 2003 and was again met with transfers and holdbacks. The crisis was not resolved until April 2003 when the legislature passed a temporary increase in the sales tax rate of an additional penny.

In summary, from FY99 to FY01, nominal GFR in Idaho grew about 10% per year on average. With the arrival of the recession from March to November 2001, the FY02 GFR was 14.5% below the Governor's forecast and there was a $280 million cyclical deficit (Barrett, et al., 2005).15 The Idaho legislature viewed the high actual and forecasted GFR of FY00, FY01, and FY02 not as aberrations but as permanent. Inadvertently, the Governor and legislature added a $65 million structural deficit to the cyclical deficit. They did this by first temporarily (FY01) and then permanently (FY02) cutting both the corporate and personal income tax rates.

Curiously, signs of a recession were already apparent in the winter of 2001 as the legislature debated the FY02 budget and tax cuts. By the summer of 2001, the Governor depleted the carryover, ordered holdbacks, and drew down the modest stabilization fund to meet FY02

[15] Our data show the gap between the 18-month forecast and actual GFR in Idaho as 18.3% for FY02 (See Table 5).

commitments. By January 2003, the Governor and legislature were in full crisis-management mode to meet an additional $205 million deficit: depleting the state building fund, tobacco settlement money, and requiring further holdbacks on state programs. It took a record-long legislative session in 2003 for the legislators to agree to the Governor's request to raise the sales tax rate temporarily from 5 to 6 cents as the way to resolve this cyclical-surplus-turned-structural-and-then-cyclical deficit crisis (Barrett, et al., 2005), (Hou, 2006).

One question remains unanswered: why did the *adding-up* approach provide such an unrealistically high forecast for the FY02 GFR by as much as $280 million? A GFR forecast is the product of an amount of GFR in the past and the rate of growth in GFR anticipated in the future. Therefore, the two prime suspects for forecast error are the rate and the base. We analyzed the rate of growth in GFR implied by the Governor's forecast of GFR. To calculate the year-to-year growth in GFR assumed by the Governor's forecast, we measured the percent change in GFR between the previous fiscal year's six-month forecast and the subsequent fiscal year's 18-month forecast. We compared this growth rate with the year-to-year *rule-of-thumb* forecast. For FY98 to FY04 these growth rates were very similar. So similar in fact that if you multiply the *rule-of-thumb* GFR forecast in the previous year by the growth rate from the *adding-up* approach, the results of both are within plus or minus $20 million dollars. This suggests that the problem with the *adding-up* FY02 forecast was not with the rate but with the base, i.e., reference year GFR to which the rate is applied.

Absent the *rule-of-thumb* comparison, how could an analyst using the *adding-up* approach have known the FY02 forecast was extraordinarily high? A way to check the reliability of the reference year GFR used in the *adding-up* forecast in FY02 is to look at the growth rates in the

preceding fiscal years FY99 to FY01. In each of these years the nominal GFR grew or was expected to grow by 10% or more (State of Idaho Division of Financial Management, 2001). But in the three years from FY96 to FY98, GFR grew at half that rate. Which is more representative of long-term growth in GFR? If we increase GFR in FY98 by 105% over each of next four years, these forecasts for FY99 to FY02 come within $20 million of the actual GFR in FY02. The Governor's predicted growth in FY02 GFR was only 2% but it was applied to a reference year GFR that had grown 31% over the previous three years. The Governor's FY02 forecast was off by about $280 million. So this problem could have been avoided with a more prudent judgment about the effect of short-term economic expansion on the level of GFR. This judgment would be better informed by using information provided by both the batch estimation and the probability density function techniques applied to the *rule-of-thumb* approach.

Summary and Conclusions

Idaho has a GFR stability problem associated with cyclical deficits. The conventional *adding-up-of-revenue-streams* approach to forecast GFR does not inform decision makers about one-time surpluses or deficits. In fact the *adding-up* forecast for FY02 paved the way for the structural and cyclical deficit that followed. In Idaho from FY70 to FY06, we found that there is a 1:1 relationship between the growth in real per capita GFR and the growth in real per capita personal income, i.e., the elasticity is 1.04 (± 0.14%). We interpret this relationship as a long-run revealed preference for government services. We used the econometrically estimated parameters of this relationship between GFR and personal income along with a batch estimation technique to develop a probability distribution of forecasts. A *rule-of thumb*

forecast is a point forecast at the mean of this probability distribution. Alternatively, a density forecast is the transformation of this distribution into a probability density function, integrated across $50 million intervals into a tabular histogram. Both the *rule-of-thumb* and density forecasts provide a long-run context for anticipating cyclical surpluses and deficits. The differences between the *rule-of-thumb* and the *adding-up* forecasts will indicate the magnitude of a budget surplus or deficit. The *adding-up* GFR forecast will fall within a density forecast, which denotes the likelihood it will occur based on the random variability of past GFR. In addition, the 95% confidence intervals for the distribution of GFR forecasts from FY07 to FY10 suggest a target range for the budget stabilization fund of about 10 to 12% of GFR, or about $290 to $350 million on a base of about $3 billion.

The *adding-up* approach does not look for long run trends in GFR but rather tries to accurately track year-to-year variability in GFR. This approach models short-run variability in GFR as if multiplying next year's growth and last year's base. In a period of extraordinary growth as existed in FY99 to FY01 simply multiplying a growth factor and the previous year's base amount of GFR is problematic because the prior year base could have been a very unusual year.

The context of the *rule-of-thumb* forecast over the course of the business cycle from FY98 to FY06 makes the problem of multiplying last year's base by next year's growth clear. This problem outlined through one business cycle is as follows. An economic expansion causes GFR to increase more than predicted, i.e., the "missed turning point" problem in forecasting. The increase in GFR creates budget surpluses in the present. Higher GFR changes the level of the reference year GFR for forecasting purposes. A higher reference year GFR results in a

forecast overstatement in the future. The surpluses and optimistic forecasts lead to tax cuts and expenditures increases setting the stage for a structural deficit. A recession begins and another turning point is missed. A lower GFR creates a combined cyclical and structural deficit. To balance the budget, the unexpected twin deficits result in depleted carryovers, transfers out of stabilization funds, negative supplementals, and tax increases. The lower GFR changes the level of the reference year GFR for forecasting purposes. The lower reference year GFR results in forecast understatements in the future. The pessimistic forecast and deficits leads to expenditure cuts and tax increases setting the stage for a structural surplus. Another economic recovery then causes an unexpected increase in GFR. The unexpected twin surpluses result in large carryovers, transfers into the stabilization funds, positive supplementals, and tax decreases. The cycle repeats making the criterion of GFR stability very difficult to meet.

The *rule-of-thumb* approach breaks this cycle through the batch estimation process that generates a probability distribution of forecasts. The mean of the forecast distribution represents a stable reference year GFR at the center of the randomly distributed variability from the past. Because the *rule-of-thumb* is not trying to measure short-run variability in GFR, there is no need to predict the base amount as being higher or lower than average. The essence of the *rule-of-thumb* approach is to multiply the mean of distribution of GFR in the past by the growth in GFR from the past to the future.

A necessary condition for preventing cyclical deficits is an improved approach to forecasting GFR (Hou, 2006). We believe the *rule-of-thumb* approach meets Hou's identified need to resolve the complexities associated with forecasting GFR in an uncertain economic

environment. The *rule-of-thumb* approach provides a long-term paradigm for budgeting. Because the mean of the forecast distribution in GFR moves 1:1 with the growth in personal income, one policy alternative is to peg the state's budgets to the expected growth in personal income as expressed by the long-term GFR trend. Year-to-year movements in GFR away from the long-run growth in personal income would signal the need to make transfers into and out of the stabilization funds. The key policy variable for legislators then would be to set the preference for the growth in GFR in relation to personal income if the current revealed preference of 1:1 ratio is not acceptable. Combined with Idaho's already sufficient stabilization fund, a long-run *rule-of-thumb* approach to GFR forecasting would make a significant contribution to stabilizing funding for public services. By using the *rule-of-thumb* GFR as the reference year GFR, the *adding-up* forecast is still helpful in anticipating short-term deficits and surpluses relative to the *rule-of-thumb* forecast. The simple data needs and straightforward algorithms make the *rule-of-thumb* approach well within the capability of state revenue analysts.

Tables

Table 1. Regression Results for Estimating the elasticity of General Fund Revenue to personal income (e_{gy})

Dependent Variable: $\ln(R_0/R_1)$ General Fund Revenue
Method: Least Squares
Sample (adjusted): 1971 2006
Included observations: 36 after adjustments
Convergence achieved after 18 iterations
Backcast: 1970

Variable	Coefficient	Std. Error	t-Statistic	Prob.
$\ln(Y_0/Y_1)$ personal income	1.04	0.07	14.76	0.00
AR(1)	0.37	0.21	1.80	0.08
MA(1)	0.62	0.17	3.71	0.00

R-squared	0.95	Mean dependent var	-0.20
Adjusted R-squared	0.95	S.D. dependent var	0.18
S.E. of regression	0.04	Akaike info criterion	-3.55
Sum squared resid	0.05	Schwarz criterion	-3.42
Log likelihood	66.96	Durbin-Watson stat	2.09

Inverted AR Roots	.37
Inverted MA Roots	-.62

Table 2. Rule-of-Thumb and Density Forecasts of Idaho GFR: FY06 to FY10

Class interval ($ mil.)		FY06 (%)	FY07 (%)	FY08 (%)	FY09 (%)	FY10 (%)
3,500	3,550					0
3,450	3,500					2
3,400	3,450					6
3,350	3,400					10
3,300	3,350				0	16
3,250	3,300				0	23
3,200	3,250				1	19
3,150	3,200				6	11
3,100	3,150			0	11	6
3,050	3,100			0	19	3
3,000	3,050			2 U_A	25	1
2,950	3,000			7	18	0
2,900	2,950			13	10	0
2,850	2,900			23	5	1
2,800	2,850		0 X	25 G_A	2	1
2,750	2,800		3	16	1	0
2,700	2,750		9	8	0	0
2,650	2,700		17	3	1	0
2,600	2,650		26	1	1	
2,550	2,600		21	0	0	
2,500	2,550	0	12	1		
2,450	2,500	4	5	2		
2,400	2,450	11 X	2	1		
2,350	2,400	22	1 U_A	0		
2,300	2,350	29	0 G_A			
2,250	2,300	18	1			
2,200	2,250	8	1			
2,150	2,200	3				
2,100	2,150	1				
2,050	2,100	0 G_A U_A				
2,000	2,050	2				
1,950	2,000	1				
Total	(%)	100	100	100	100	100
--- Top 2 intervals	"	51	48	48	43	42
Point forecast						
RoT	($ mil.)	2,322	2,602	2,829	3,014	3,254
Gov. G_A	"	2,090	2,308	2,808		
Unv. U_A	"	2,097	2,383	3,022		
Actual X	"	2,431	2,813			
Predicted surplus or deficit						
G_A -RoT	($ mil.)	-232	-294	-21		
U_A -RoT	"	-225	-219	193		
Actual surplus or deficit						
X - RoT	($ mil.)	110	73			

G_A and U_A are 18-month forecasts of the Governor's and university economists respectively.

Figures

Figure 1. Growth in Idaho's General Fund Revenue and Personal Income: FY70 to FY06 (real per capita, avg. FY02 to FY06 base)

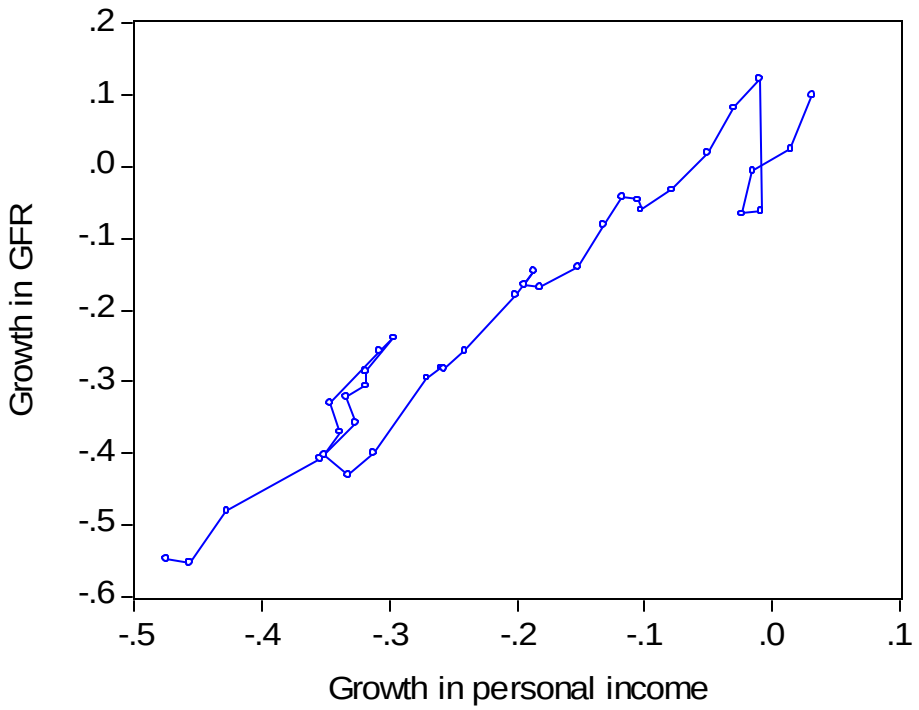
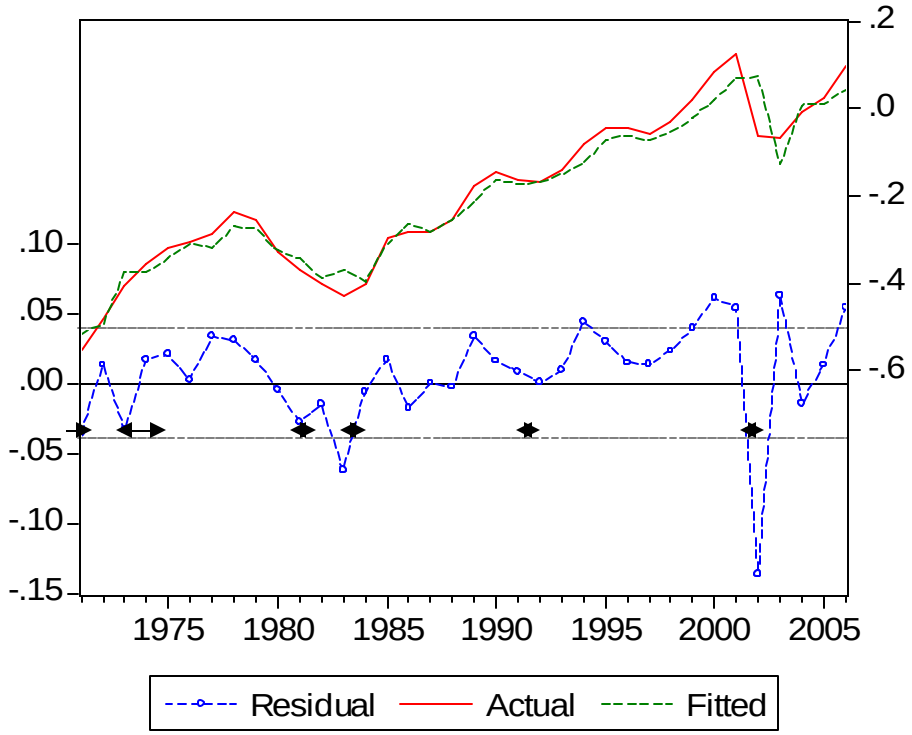
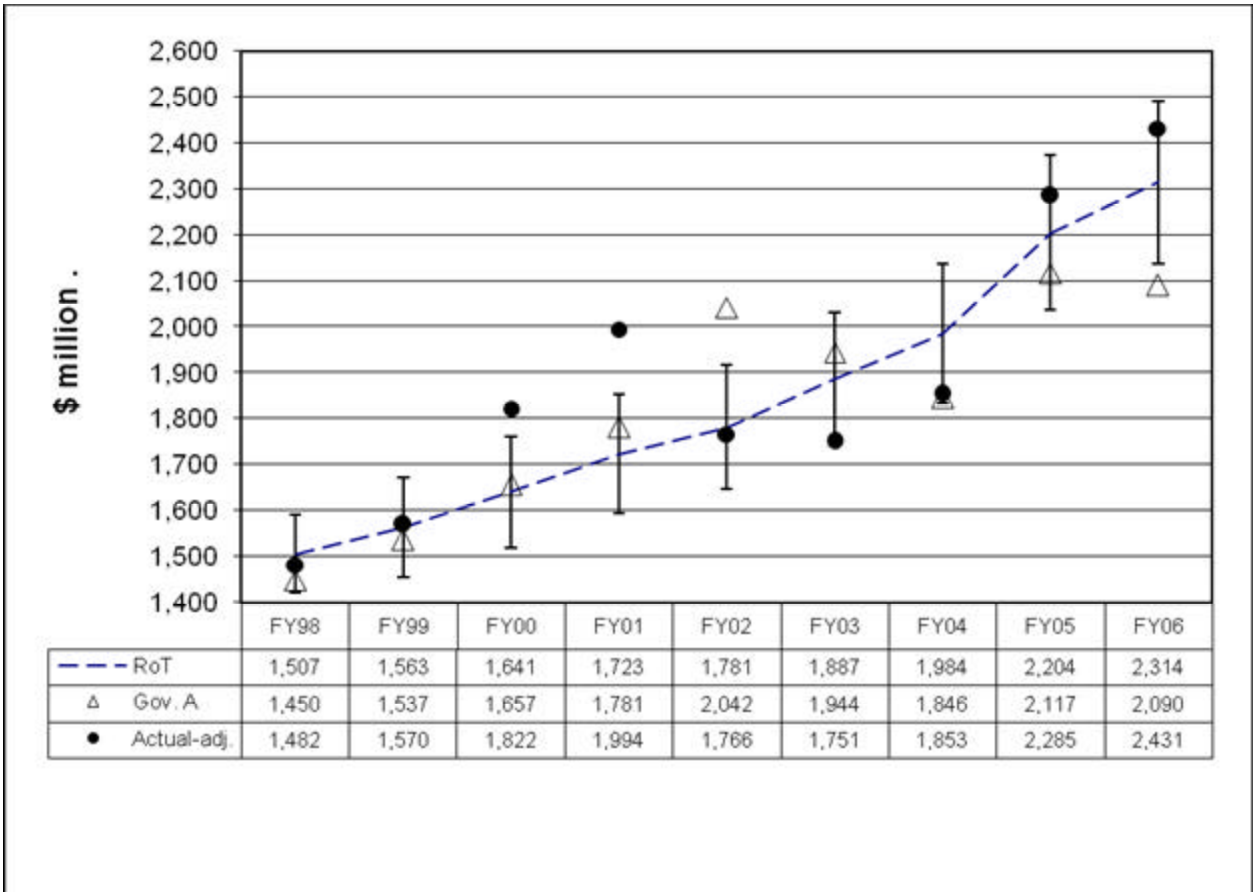


Figure 2. Fitted Equation to the Actual Growth in General Fund Idaho: FY70 to FY06



↔ Beginning and end of downturns in the U.S. business cycle

Figure 3. Actual and Predicted nominal General Fund Revenue in Idaho: FY98 to FY06



	FY98	FY99	FY00	FY01	FY02	FY03	FY04	FY05	FY06
RoT	1,507	1,563	1,641	1,723	1,781	1,887	1,984	2,204	2,314
Gov. A	1,450	1,537	1,657	1,781	2,042	1,944	1,846	2,117	2,090
Actual-adj.	1,482	1,570	1,822	1,994	1,766	1,751	1,853	2,285	2,431

Note: "Actual-adj." refers to the actual GFR plus or minus revenue resulting from tax code modifications made subsequent to the 18-month (A) forecast.

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Appendices

Appendix 1. Mathematics of the Model

A1.1 A Model of General Fund Revenue

Assume that nominal General Fund revenue (R_N) is an increasing function of inflation (C), population (P), sales tax rate (S) and real per capita personal income (Y)

1) $R_N = R(C,P,S,Y), \qquad \left(R_C' > 0,\ R_P' > 0,\ R_S' > 0,\ R_Y' > 0\right)$

and that C, P, S, and Y are functions of time (t)

$$C = C(t), \qquad P = P(t), \qquad S = S(t), \qquad Y = Y(t)$$

Taking a logarithmic transformation of (1)

2) $\ln R_N = \ln R(\ln C, \ln P, \ln S, \ln Y)$

$$\ln C = \ln C(t), \quad \ln P = \ln P(t), \quad \ln S = \ln S(t), \quad \ln Y = \ln Y(t)$$

To know how R_N changes over time, take the differential of R with respect to t:

3) $$d\ln R_N = \left(\frac{\partial \ln R}{\partial \ln C}\right) d\ln C + \left(\frac{\partial \ln R}{\partial \ln P}\right) d\ln P + \left(\frac{\partial \ln R}{\partial \ln S}\right) d\ln S + \left(\frac{\partial \ln R}{\partial \ln Y}\right) d\ln Y$$

$$d\ln C = \frac{1}{C}\frac{dC}{dt}dt, \qquad d\ln P = \frac{1}{P}\frac{dP}{dt}dt, \qquad d\ln S = \frac{1}{S}\frac{dS}{dt}dt, \qquad d\ln Y = \frac{1}{Y}\frac{dY}{dt}dt,$$

4) $$\frac{1}{R_N}dR = \left(\frac{C}{R_N}\frac{\partial R}{\partial C}\right)\frac{1}{C}\frac{dC}{dt}dt + \left(\frac{P}{R_N}\frac{\partial R}{\partial P}\right)\frac{1}{P}\frac{dP}{dt}dt + \left(\frac{S}{R_N}\frac{\partial R}{\partial S}\right)\frac{1}{S}\frac{dS}{dt}dt + \left(\frac{Y}{R_N}\frac{\partial R}{\partial Y}\right)\frac{1}{Y}\frac{dY}{dt}dt$$

5) $$\frac{1}{R_N}\frac{dR}{dt} = \left(\frac{C}{R_N}\frac{\partial R}{\partial C}\right)\frac{1}{C}\frac{dC}{dt} + \left(\frac{P}{R_N}\frac{\partial R}{\partial P}\right)\frac{1}{P}\frac{dP}{dt} + \left(\frac{S}{R_N}\frac{\partial R}{\partial S}\right)\frac{1}{S}\frac{dS}{dt} + \left(\frac{Y}{R_N}\frac{dR}{dY}\right)\frac{1}{Y}\frac{dY}{dt}$$

6) $r_{R_N} = e_{RC}\, r_C + e_{RP}\, r_P + e_{RS}\, r_S + e_{RY}\, r_Y$

Define real per capita sales-tax-rate-adjusted General Fund revenues as:

7) $r_R \equiv r_{R_N} - e_{RC}\, r_C - e_{RP}\, r_P - e_{RS}\, r_S$

8) $r_{R_N} = r_R + e_{RC}\, r_C + e_{RP}\, r_P + e_{RS}\, r_S$

A similar algorithm is used to define both real and nominal per capita personal income.

9) $r_Y \equiv r_{Y_N} - e_{YC}\, r_C - e_{YP}\, r_P$

10) $r_{Y_N} = r_Y + e_{YC}\, r_C + e_{YP}\, r_P$

By substituting (7) into (6) results in the growth in adjusted real per capita GFR (eq.11) equal to the growth in real per capita personal income weighted by the elasticity of revenue to income.

11) $r_R = e_{RY}\, r_Y$

r_{R_N} is the rate of growth in nominal General Fund revenue

r_R is the rate of growth in sales tax adjusted real per capita General Fund revenue

r_C is the rate of growth in inflation

r_P is the rate of growth in population

r_S is the rate of growth in the sales tax rate

r_{Y_N} is the rate of growth in nominal personal income

r_Y is the rate of growth in real per capita personal income, and

e_{RX} is the elasticity of revenue to x (inflation, population, sales tax rate, or income)

e_{YX} is the elasticity of income to x (inflation and population).

(Chiang and Wainwright, *Mathematical Economics*, 4th ed., p. 290)

A1.2 A Model of the Growth in General Fund Revenue

Let the specific function for continuous growth be:

12) $$Z_1 = Z_i e^{tr_Z}$$

where Z is an arbitrary variable

subscript i is an arbitrarily chosen base period against which growth in period 1 is compared.

By logarithmic transformation, this equation is:

13) $$\ln Z_1 = \ln Z_i + tr_Z$$

Solving for the rate of growth (r_Z)

14) $$r_Z = \frac{\ln\left(\frac{Z_1}{Z_i}\right)}{t}$$

Taking the derivative of equation (13) with respect to time (t)

15) $$\frac{1}{Z_1}\frac{dZ}{dt} = r_Z = \frac{\ln\left(\frac{Z_1}{Z_i}\right)}{t}$$

By applying the results of equations (15) to equation (6),

we get a specific equation in logs to predict the growth in nominal GFR:

16) $$\ln\left(\frac{R_{1N}}{R_{iN}}\right) = (e_{RC})\ln\left(\frac{C_1}{C_i}\right) + (e_{RP})\ln\left(\frac{P_1}{P_i}\right) + (e_{RS})\ln\left(\frac{S_1}{S_i}\right) + (e_{RY})\ln\left(\frac{Y_1}{Y_i}\right)$$

The equation for General Fund revenue as an exponential growth function is:

17) $$\frac{R_{1N}}{R_{iN}} = \left(\frac{C_1}{C_i}\right)^{(e_{RC})}\left(\frac{P_1}{P_i}\right)^{(e_{RP})}\left(\frac{S_1}{S_i}\right)^{(e_{RS})}\left(\frac{Y_1}{Y_i}\right)^{(e_{RY})}$$

Converting nominal to adj. real percapita General Fund revenes is accomplished by:

18) $$\frac{R_1}{R_i} \equiv \left(\frac{R_{1N}}{R_{iN}}\right)\left(\frac{C_1}{C_i}\right)^{-(e_{RC})}\left(\frac{P_1}{P_i}\right)^{-(e_{RP})}\left(\frac{S_1}{S_i}\right)^{-(e_{RS})}$$

19) $$\frac{R_{1N}}{R_{iN}} = \left(\frac{R_1}{R_i}\right)\left(\frac{C_1}{C_i}\right)^{(e_{RC})}\left(\frac{P_1}{P_i}\right)^{(e_{RP})}\left(\frac{S_1}{S_i}\right)^{(e_{RS})}$$

Applying a similar algorithm transforms nominal to real percapita personal income

20) $$\frac{Y_1}{Y_i} \equiv \left(\frac{Y_{1N}}{Y_{iN}}\right)\left(\frac{C_1}{C_i}\right)^{-(e_{YC})}\left(\frac{P_1}{P_i}\right)^{-(e_{YP})}$$

Substituting (18) into (17), eq. (21) is the equation used

to isolate the parameter (e_{RY}).

21) $$\frac{R_1}{R_i} = \left(\frac{Y_1}{Y_i}\right)^{(e_{RY})}$$

A1.3 Probability Distribution for a Forecast of General Fund Revenue

The fitted equation used to estimate the growth in real GFR with the present (R_1 and Y_1) as the base year relative to the past (R_i and Y_i) is :

22) $$-\ln\left(\frac{R_i}{R_1}\right) = -e_{RY}\ln\left(\frac{Y_i}{Y_1}\right) \Rightarrow \left(\frac{R_i}{R_1}\right)^{-1} = \left(\frac{Y_i}{Y_1}\right)^{-e_{RY}}$$

To predict GFR with the future ($\hat{R}_j$ and $\hat{Y}_j$) as the base year relative to the present (R_1 and Y_1) the predicted equation is :

23) $$\frac{\hat{R}_j}{R_1} = \left(\frac{\hat{Y}_j}{Y_1}\right)^{e_{RY}} \Rightarrow \left(\frac{R_1}{\hat{R}_j}\right)^{-1} = \left(\frac{Y_1}{\hat{Y}_j}\right)^{-e_{RY}}$$

To reconcile the fitted and predicted equations in which estimated growth from any past observatio n can serve as the reference year to predict GFR in the future, the equation is :

24) $$\left(\frac{R_i}{R_1}\right)^{-1}\left(\frac{R_1}{\hat{R}_j}\right)^{-1} = \left(\frac{Y_i}{Y_1}\right)^{-e_{RY}}\left(\frac{Y_1}{\hat{Y}_j}\right)^{-e_{RY}}$$

25) $$\frac{\hat{R}_j}{R_i} = \left(\frac{\hat{Y}_j}{Y_i}\right)^{e_{RY}}$$

The equation to forecast nominal GFR is :

26) $$\hat{R}_{Nj} = R_i\left(\frac{\hat{Y}_j}{Y_i}\right)^{e_{RY}}\left(\frac{\hat{C}_j}{C_1}\right)^{(e_{RC})}\left(\frac{\hat{P}_j}{P_1}\right)^{(e_{RP})}\left(\frac{\hat{S}_j}{S_1}\right)^{(e_{RS})}$$

By assuming $e_{RC} = e_{RP} = e_{RS} = 1$, the rule - of - thumb equation of the growth in nominal GFR that incorporates the fitted parameter e_{RY} and outside forecasts of Y, C, P, and S is :

27) $$\hat{R}_{Nj} = R_i\left(\frac{Y_i}{Y_1}\right)^{-e_{RY}}\left(\frac{\hat{Y}_j}{Y_1}\right)^{e_{RY}}\left(\frac{\hat{C}_j}{C_1}\right)\left(\frac{\hat{P}_j}{P_1}\right)\left(\frac{\hat{S}_j}{S_1}\right)$$

For added stability of the forecast,if we repeatedly change the reference year i, then we can generate a point forecast of GFR as the mean of the batch forecasts in year j:

28) $$\overline{\hat{R}}_{Nj} = \frac{1}{n}\sum_{i=1}^{n}\left(R_i\left(\frac{Y_i}{Y_1}\right)^{-e_{RY}}\left(\frac{\hat{Y}_j}{Y_1}\right)^{e_{RY}}\left(\frac{\hat{C}_j}{C_1}\right)\left(\frac{\hat{P}_j}{P_1}\right)\left(\frac{\hat{S}_j}{S_1}\right)\right)$$

where i represents the reference years of FY71 through FY06.

For added clarity, we can also calculate the mean of the confidence interval forecasts around the point forecast of GFR :

29) $$\overline{\hat{R}}_{Nj} \pm c.i. = \frac{1}{n}\sum_{i=1}^{n}\left(R_i\left(\frac{Y_i}{Y_1}\right)^{-e_{RY}}\left(\frac{\hat{Y}_j}{Y_1} \pm 2 * s.e.\right)^{e_{RY}}\left(\frac{\hat{C}_j}{C_1}\right)\left(\frac{\hat{P}_j}{P_1}\right)\left(\frac{\hat{S}_j}{S_1}\right)\right)$$

where s.e. is the forecast standard error.

A1.4 Density Forecasts of General Fund Revenue

Each observation ($\hat{R}_{N.n}$) is a prediction within an assumed normal distribution of nominal GFR forecasts for a given fiscal year associated with a reference year between 1971 and 2006, n = 35.

The Gaussian probability density function of the nominal GFR forecast is:

30) $$f\left(\hat{R}\right) = \frac{1}{n}\left(\sum_{i=1}^{n} \frac{1}{h\sqrt{2\mathrm{p}}} \exp\left(-\frac{\left(R - \hat{R}_{N.n}\right)^2}{2h^2}\right)\right)$$

Each prediction $\hat{R}_{N.n}$ is an expected value of the mean and h is its standard deviation.

31) $$h = \min \begin{cases} h_1 = 1.06 s\, n^{-0.2} \\ h_2 = 0.9 \dfrac{Q}{1.34} n^{-0.2} \end{cases}$$

where s is the standard deviation across all observations of $\hat{R}_{N.n}$ and Q is the interquartile range of the distribution of $\hat{R}_{N.n}$ forecasts.

Eq. (30) results in a smoothed histogram of predictions of GFR for one year.

Eq. (31) is the algorithm to infer the standard deviation of the observation based on information about the population as a whole.

The discrete density forecast probability as a percent is

32) $$P\left(f\left(\hat{R}\right)\right) = \int_{R_1}^{R_2} f\left(\hat{R}\right) dR\left(100\right) = \Pr\left[R_1 < \hat{R} < R_2\right]$$

where the interval of integration R_1 and R_2 is a class interval in the range of potential GFR values.

Eq. (32) calculates the probability that the predicted GFR will fall within a given range based on the experience of the past 35 years.

Silverman, B. W. *Density estimation for statistics and data analysis*. London ; New York : Chapman and Hall, 1986.

Appendix 2. Nominal Data for Idaho General Fund Revenue Projections

Fiscal Year	GFR (w/o sales tax)	GFR (sales tax only)	GFR (R_N)	Personal Income (Y_N)	Population (P)	CPI (C)	Sales Tax Rate (S)
		($ million – nominal)			(#)	(1983=100)	(¢/$)
FY70	65	33	98	2,411	717,255	38.8	3.00
FY71	72	34	106	2,639	738,749	40.5	3.00
FY72	86	37	123	2,898	763,229	41.8	3.00
FY73	101	43	144	3,394	782,061	44.4	3.00
FY74	127	49	176	4,002	807,973	49.3	3.00
FY75	148	57	205	4,467	831,981	53.8	3.00
FY76	163	64	227	4,938	856,979	56.9	3.00
FY77	179	74	252	5,421	883,446	60.6	3.00
FY78	210	84	294	6,149	910,962	65.2	3.00
FY79	240	91	331	6,931	932,636	72.6	3.00
FY80	261	96	357	7,695	947,983	82.4	3.00
FY81	290	98	388	8,683	962,204	90.9	3.00
FY82	306	99	405	9,215	973,719	96.5	3.00
FY83	299	115	415	9,772	981,866	100.0	3.25
FY84	307	188	496	10,490	990,841	103.9	4.50
FY85	349	200	549	11,366	994,052	107.6	4.04
FY86	359	212	571	11,666	990,222	109.6	4.16
FY87	373	259	632	12,052	984,997	113.6	5.00
FY88	417	259	675	12,765	985,661	118.3	5.00
FY89	484	289	773	14,047	994,422	124.0	5.00
FY90	538	319	857	15,290	1,012,384	130.7	5.00
FY91	566	336	902	16,271	1,041,316	136.2	5.00
FY92	588	364	952	17,465	1,071,685	140.3	5.00
FY93	641	403	1,043	19,169	1,108,768	144.5	5.00
FY94	721	453	1,173	20,710	1,145,140	148.2	5.00
FY95	807	482	1,288	22,231	1,177,322	152.4	5.00
FY96	888	463	1,351	23,663	1,203,083	156.9	5.00
FY97	915	477	1,392	24,785	1,228,520	160.5	5.00
FY98	986	497	1,482	26,291	1,252,330	163.0	5.00
FY99	1,036	589	1,625	28,140	1,275,674	166.6	5.00
FY00	1,194	628	1,822	30,267	1,299,610	172.2	5.00
FY01	1,337	647	1,985	32,262	1,321,309	177.0	5.00
FY02	1,043	657	1,700	33,374	1,343,124	179.9	5.00
FY03	1,064	700	1,764	34,204	1,366,332	184.0	5.00
FY04	1,211	886	2,097	35,910	1,385,200	188.9	6.00
FY05	1,317	951	2,268	38,854	1,407,225	195.3	6.00
FY06	1,551	881	2,431	42,001	1,448,125	201.7	5.08

(State of Idaho Division of Financial Management, 1970 - 2007b)

Appendix 3. Deflated Data for Idaho's General Fund Revenue Predictions

Fiscal year	real GFR[a] (R_0)	real pers. income[b] (Y_0)	Actual GFR[c] $\ln(R_0/R_1)$	Actual pers. inc.[c] $\ln(Y_0/Y_1)$	Fitted GFR[d]	Residual GFR
	($ million - real)			(Percent)		(Percent)
FY70	1,274	25,302	-55	-48	-50	-5
FY71	1,268	25,763	-55	-46	-52	-4
FY72	1,361	26,533	-48	-43	-49	1
FY73	1,466	28,550	-41	-35	-37	-3
FY74	1,541	29,344	-36	-33	-37	2
FY75	1,597	29,146	-32	-33	-34	2
FY76	1,622	29,580	-31	-32	-31	0
FY77	1,655	29,577	-29	-32	-32	3
FY78	1,734	30,240	-24	-30	-27	3
FY79	1,702	29,899	-26	-31	-27	2
FY80	1,583	28,774	-33	-35	-33	0
FY81	1,521	28,997	-37	-34	-34	-3
FY82	1,472	28,643	-40	-35	-39	-1
FY83	1,432	29,187	-43	-33	-37	-6
FY84	1,476	29,762	-40	-31	-39	-1
FY85	1,639	31,038	-29	-27	-31	2
FY86	1,663	31,397	-28	-26	-26	-2
FY87	1,661	31,460	-28	-26	-28	0
FY88	1,702	31,976	-26	-24	-25	0
FY89	1,841	33,274	-18	-20	-21	3
FY90	1,903	33,751	-15	-19	-16	2
FY91	1,868	33,509	-16	-19	-17	1
FY92	1,860	33,927	-17	-18	-17	0
FY93	1,914	34,946	-14	-15	-15	1
FY94	2,032	35,644	-8	-13	-12	4
FY95	2,110	36,189	-4	-12	-7	3
FY96	2,102	36,616	-5	-11	-6	2
FY97	2,073	36,715	-6	-10	-7	1
FY98	2,132	37,619	-3	-8	-6	2
FY99	2,246	38,674	2	-5	-2	4
FY00	2,391	39,504	8	-3	2	6
FY01	2,492	40,293	12	-1	7	5
FY02	2,068	40,344	-6	-1	7	-14
FY03	2,062	39,739	-7	-2	-13	6
FY04	2,189	40,085	-1	-2	1	-1
FY05	2,255	41,294	2	1	1	1
FY06	2,431	42,001	10	3	4	5
Base[c]	2,201	40,692	0	0		
Mean						0.8

a) Adjusted for changes in inflation, population, and sales tax rate
b) Adjusted for changes in inflation and population
c) Growth in GFR (R_1) and personal income (Y_1) relative to their base, i.e., the average of FY02 to FY06